CS 61A Topical Review

Tail Recursion and Macros

Albert Xu

Slides: albertxu.xyz/teaching/cs61a/

The Cost of Recursion

ain't no free lunch

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  (if (= n 0)
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    (* n (factorial (- n 1)))))
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scm> (factorial 3)
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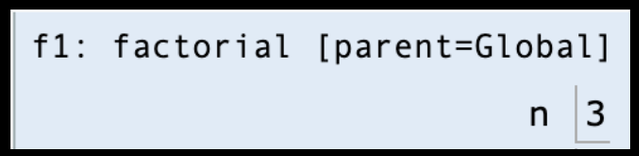

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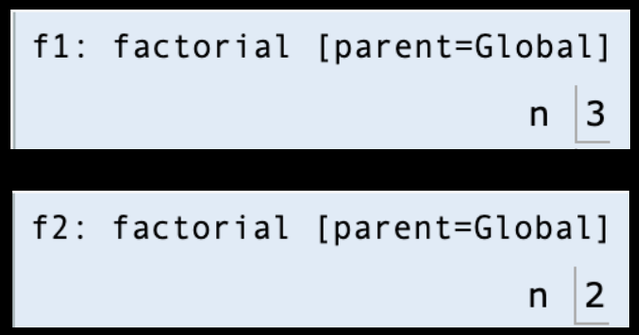

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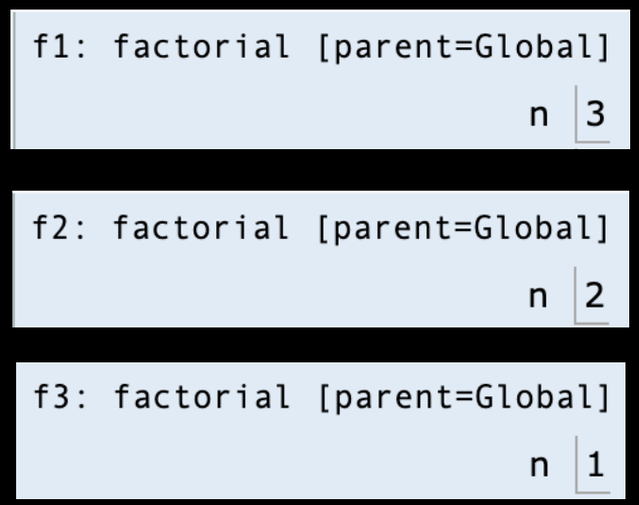

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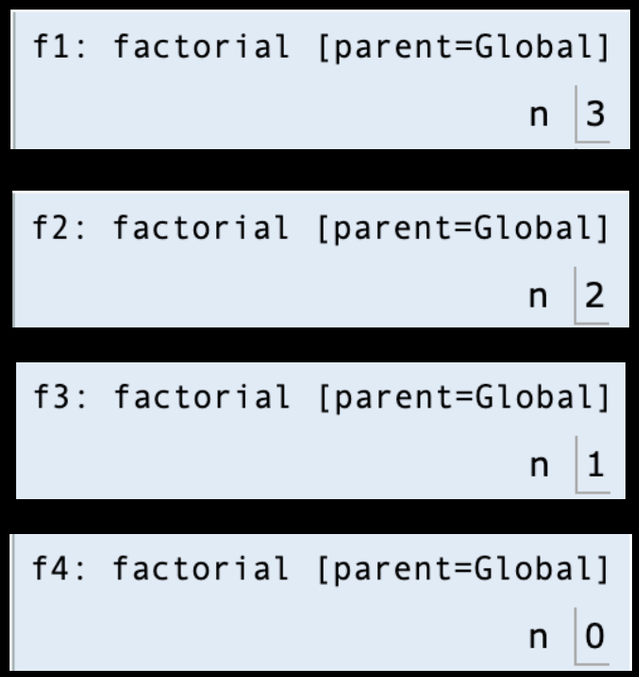

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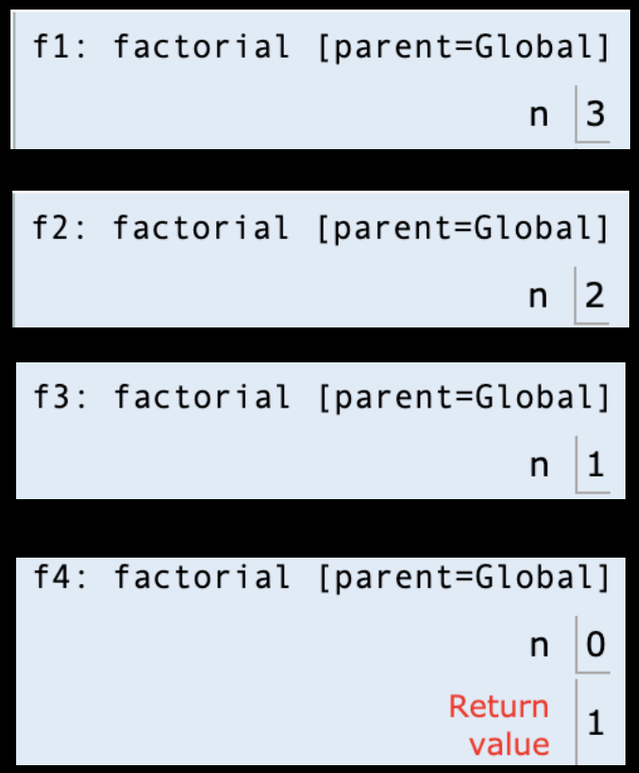

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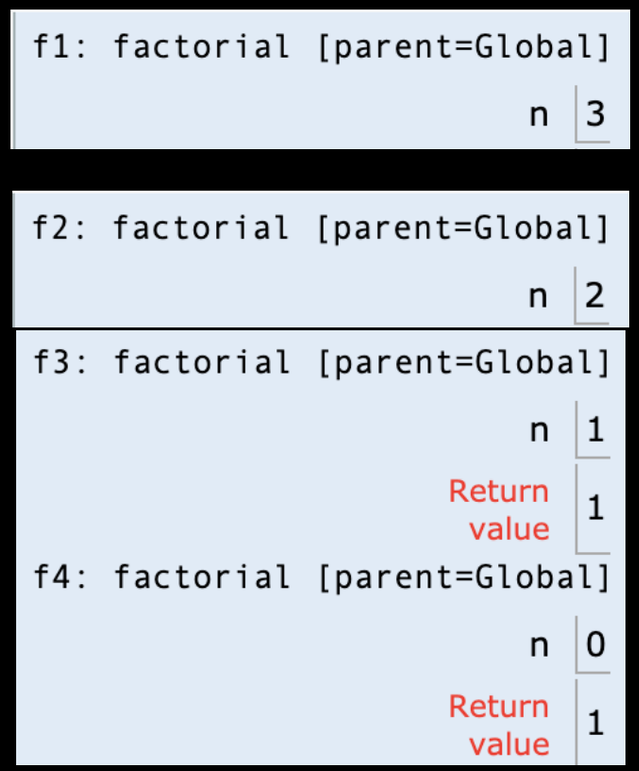

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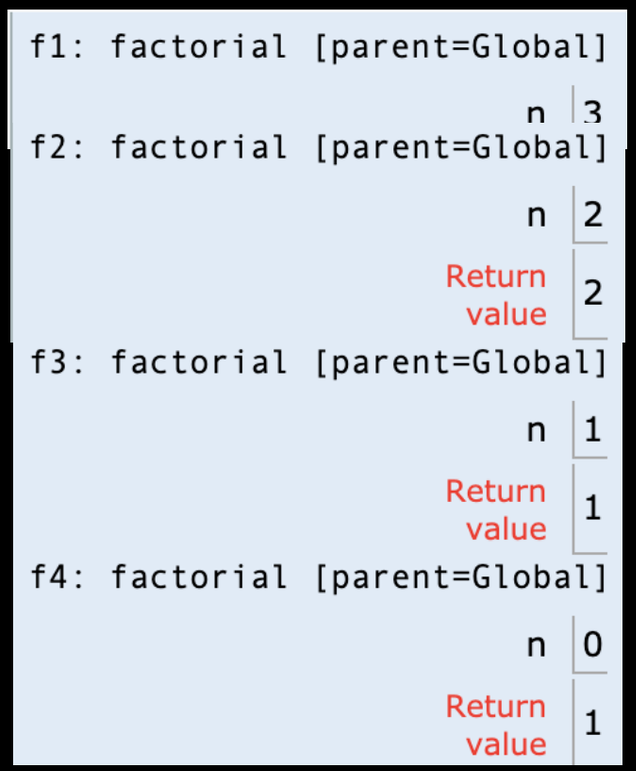

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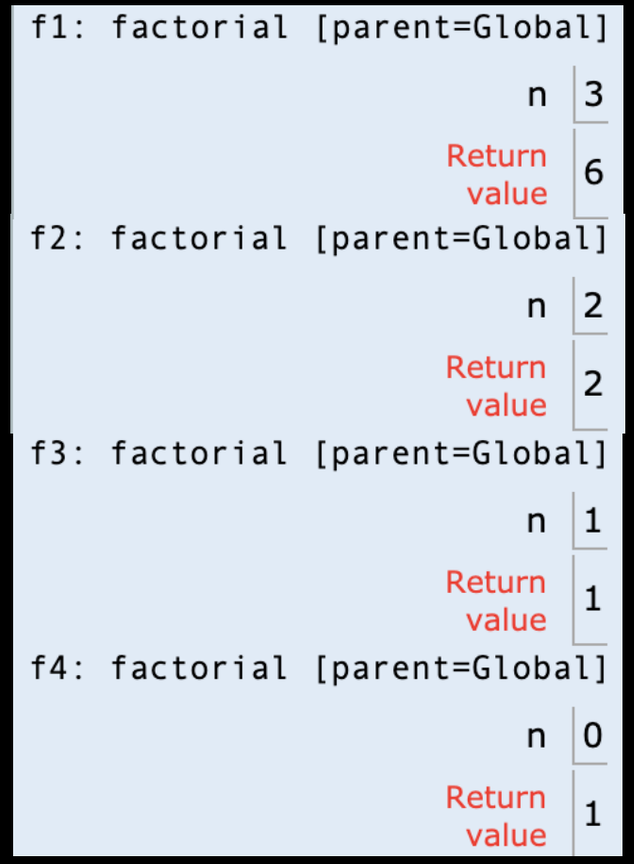

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```
f1: factorial [parent=Global]
                        n | 3
                   Return | 6
                    value |
f2: factorial [parent=Global]
                        n | 2
                   Return | 2
                    value |
f3: factorial [parent=Global]
                        n | 1
                   Return | 1
                    value |
f4: factorial [parent=Global]
                        n | 0
                   Return | 1
                    value |
```

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scm> (factorial 10)
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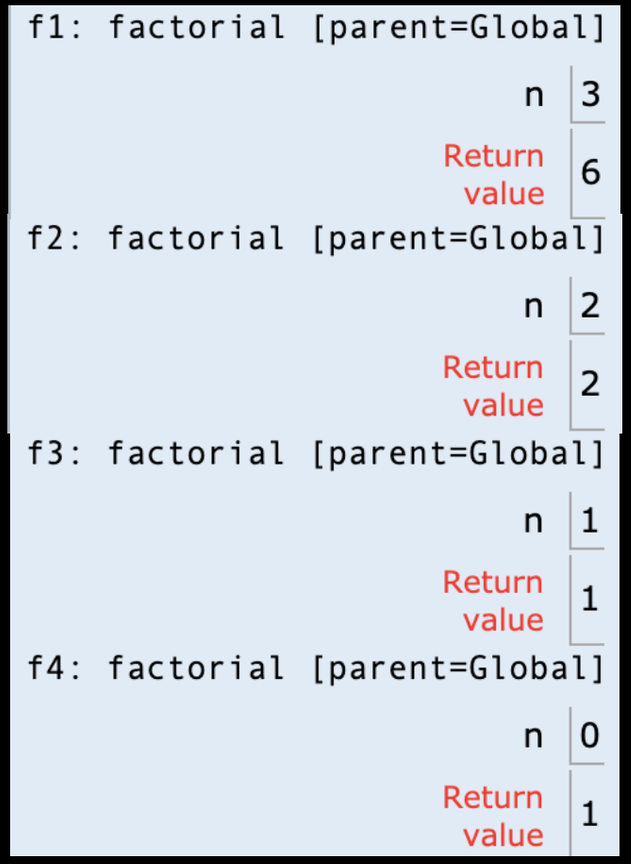


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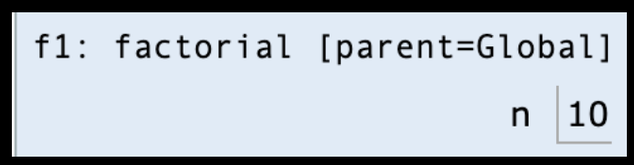

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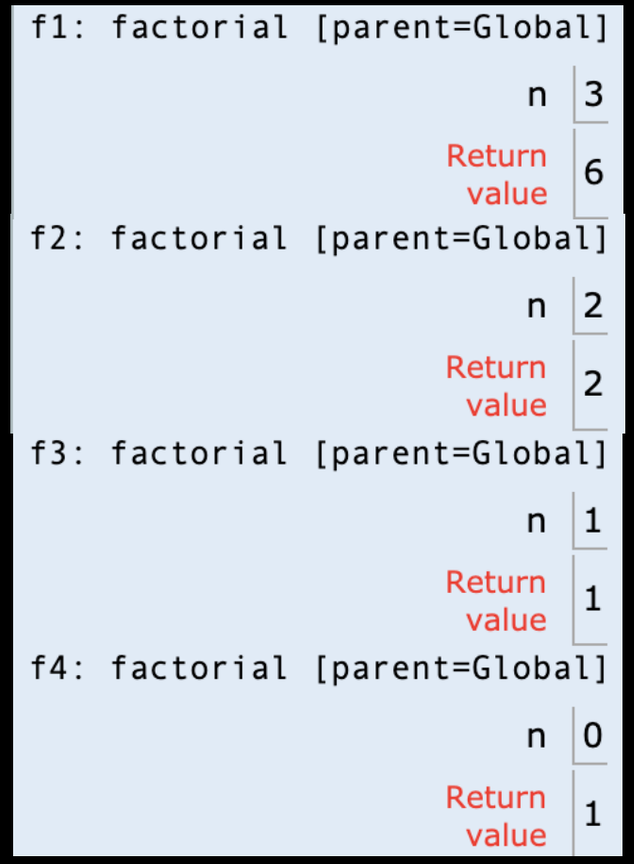


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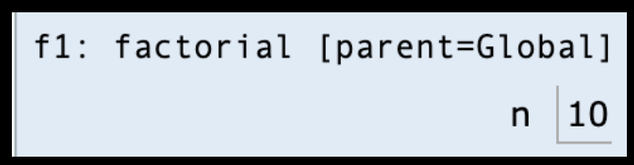

...11 frames!

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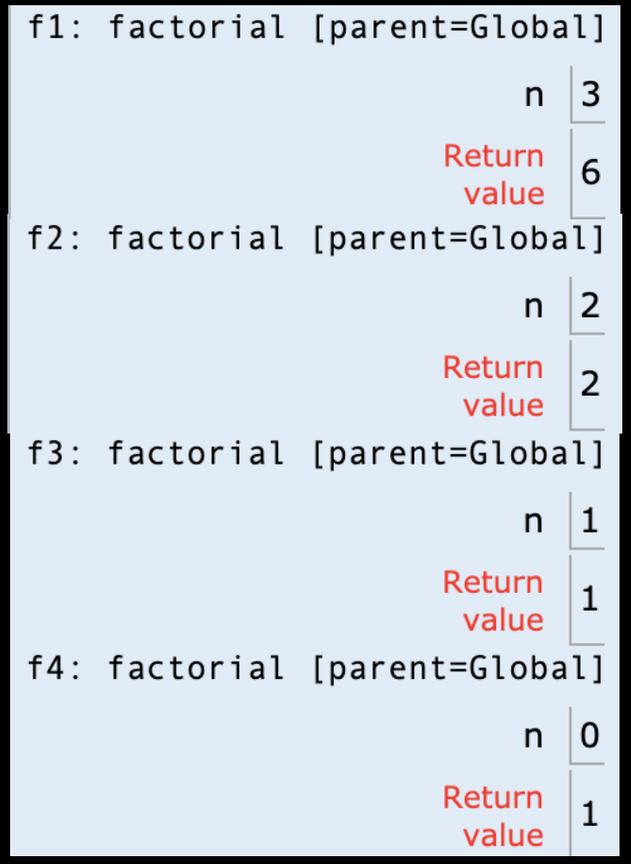


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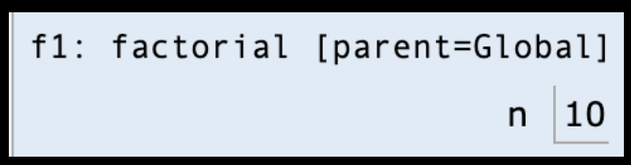

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How much memory does `factorial` take? Use big-theta notation!

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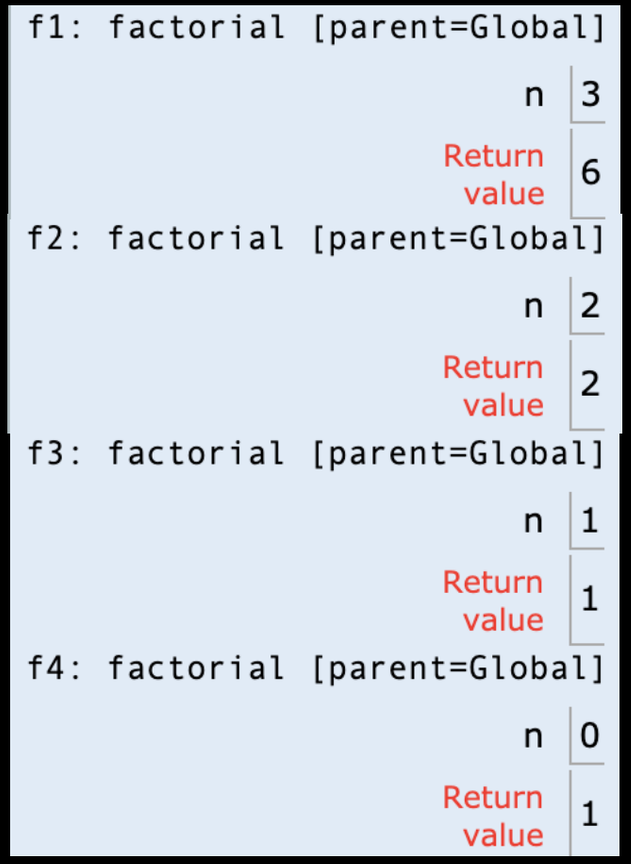


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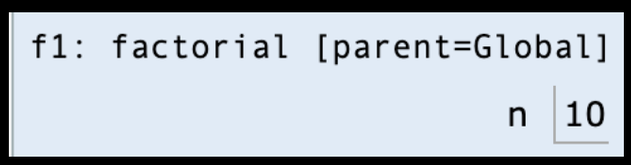

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How much memory does `factorial` take? Use big-theta notation! `θ(n)`

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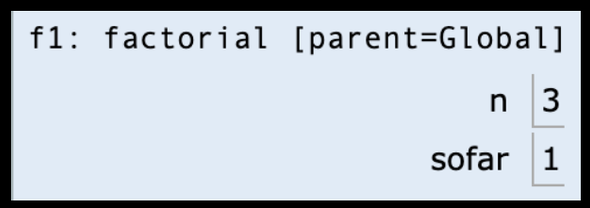

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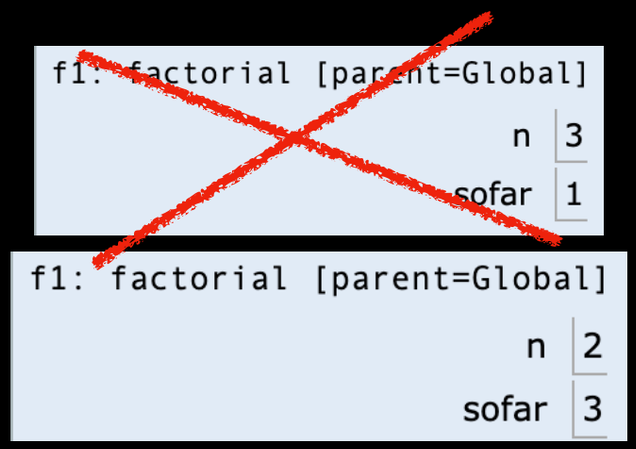

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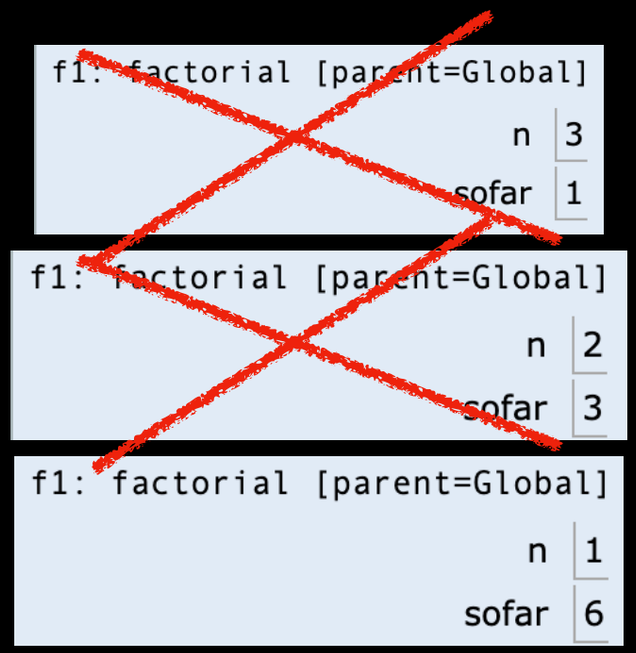

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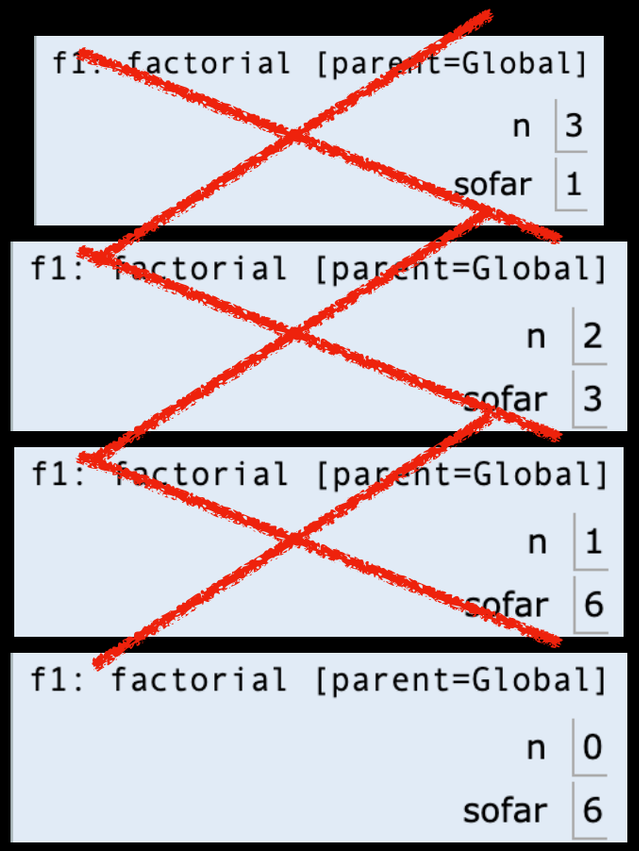

How much memory does `factorial` take now? Use big-theta notation!

`θ(1)`

Is Tail Call Optimization Worth it?

Discuss: what are the benefits and what are the drawbacks of tail-call optimization?

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Because of this drawback, Python does not perform tail-call optimization.

Tail Contexts

Tail contexts are locations in Scheme expressions where recursive calls would be the last operation performed in a frame! Why is this important?

…recursive calls in tail contexts are called tail calls!

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Question:
What makes a Scheme function tail recursive - in the context of tail calls? Check out this example if you're not sure…

Identifying Tail Contexts

How can you tell what is a tail context?

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Given that each of the following expressions is the last expression in the body of the function, the following expressions are tail contexts:

- the second or third operand in an `if` expression
- any of the non-predicate sub-expressions in a `cond` expression (i.e. the second expression of each clause)
- the last operand in an `and` or an `or` expression
- the last operand in a `begin` expression's body
- the last operand in a `let` expression's body

...some examples