CS 61A Discussion 3

Recursion and Tree Recursion

Albert Xu

Attendance: links.cs61a.org/albert-disc
Slides: albertxu.xyz/teaching/cs61a/

Announcements

Wednesday, July 10

- Small Group Tutoring Section Sign-ups are open! See Piazza @220 for enrollment details.
- Project 2, Typing Test is released.
  - Phase 1/2 due Friday, July 12th at 11:59 PM PDT.
  - Entire project is due Tuesday, July 23rd at 11:59 PM PDT. You may complete all phases with *one* partner.
  - Can get 1 point of extra credit by submitting by Monday, July 22nd at 11:59 PM PDT and filling out a feedback survey
- HW3 will be released tonight and will be due Tuesday, July 16th at 11:59 PM PDT.
- Midterm and Final will be held 6-9 PM PDT (except for W61A students the final remotely)
  - Conflicts? Fill out links.cs61a.org/mt-conflict or links.cs61a.org/final-conflict by 7/9 at 11:59 PM PDT

*Albert's extra OH next week 6-7pm
TuTh

Recursion: an example

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1  def factorial(n):
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Here are the two key parts of recursion:

1) Base case
2) Recursive call

Look for them in this example.

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Base case

Recursive call (and multiply it by n)

This function works because n! = n * (n - 1)!,
which is our recursive call.

Recursion: dominoes

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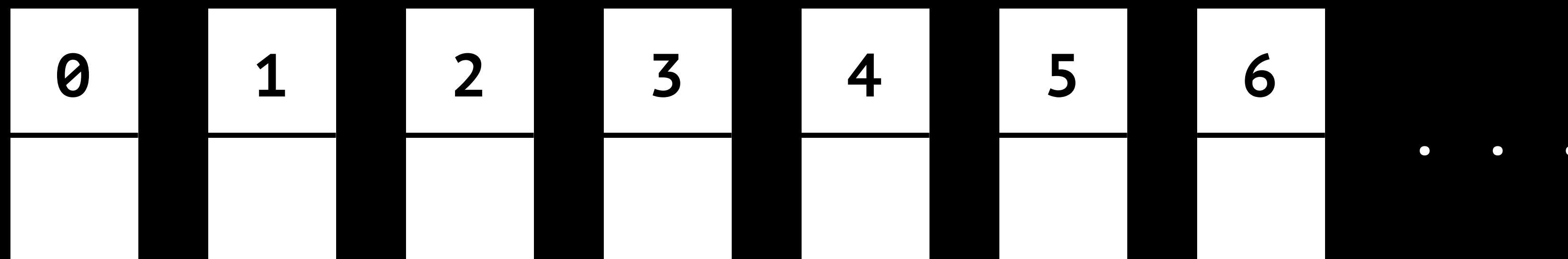

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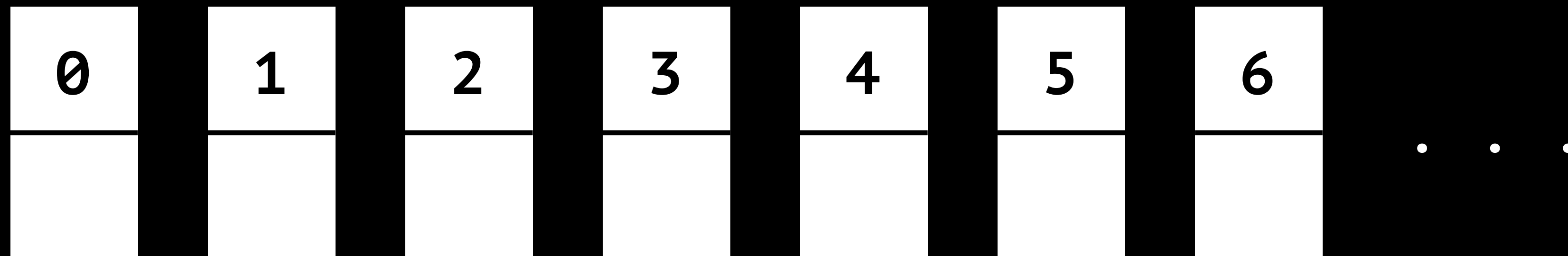

Dominoes again. Remember induction? Well think of this as induction - but backwards.

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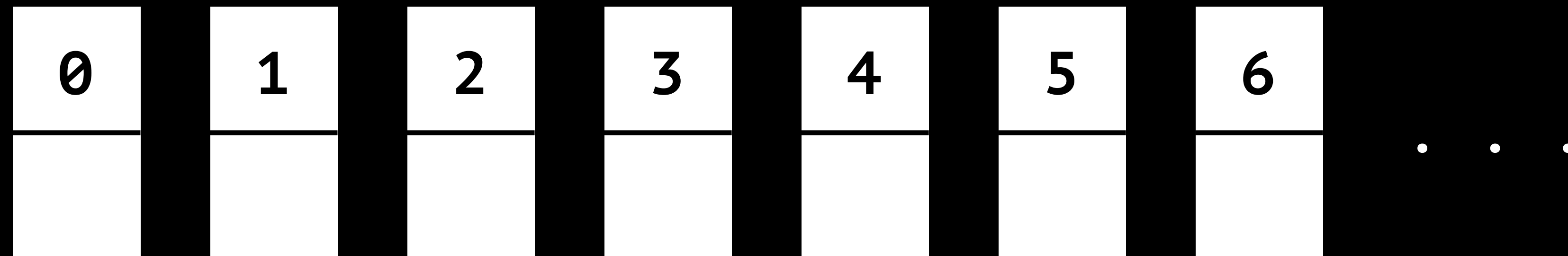
Let's calculate factorial(4)!



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Recursion: dominoes

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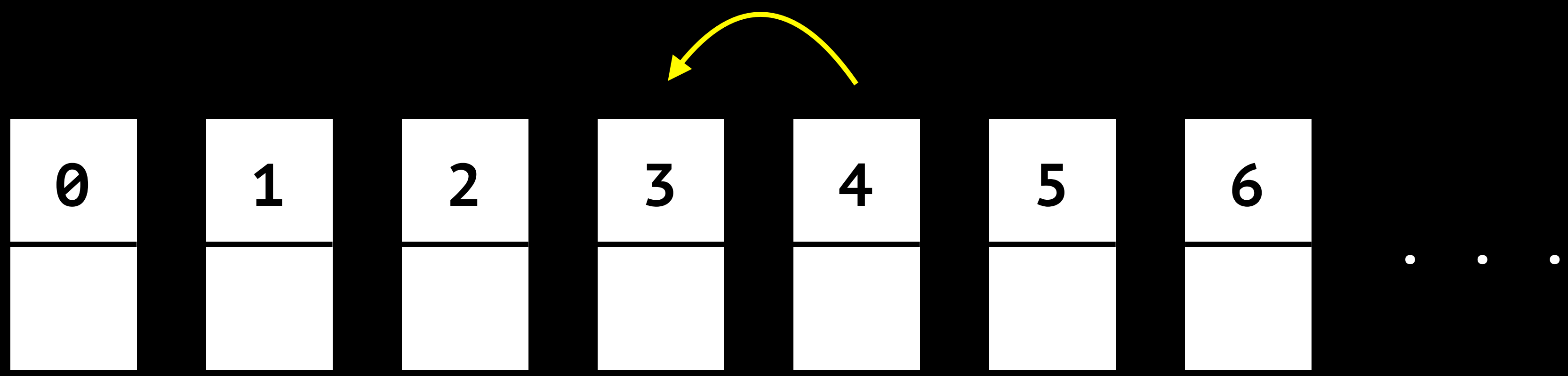
factorial(4) = 4 * factorial(3)



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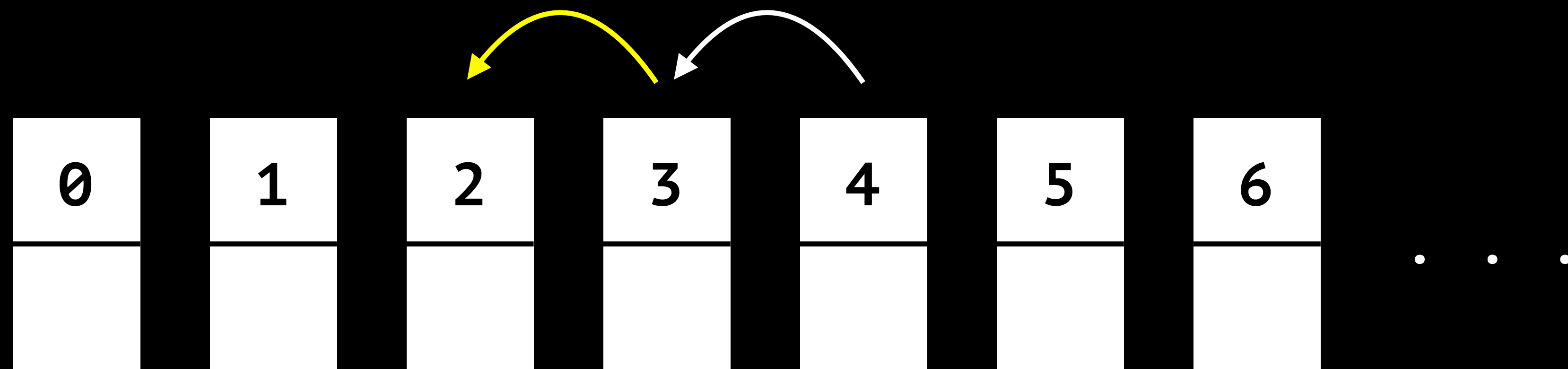

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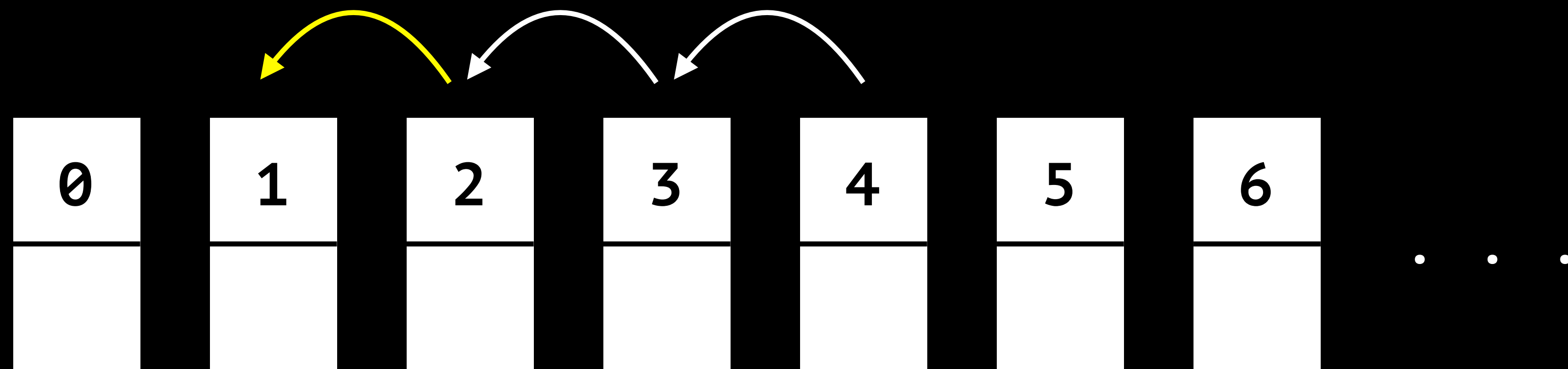

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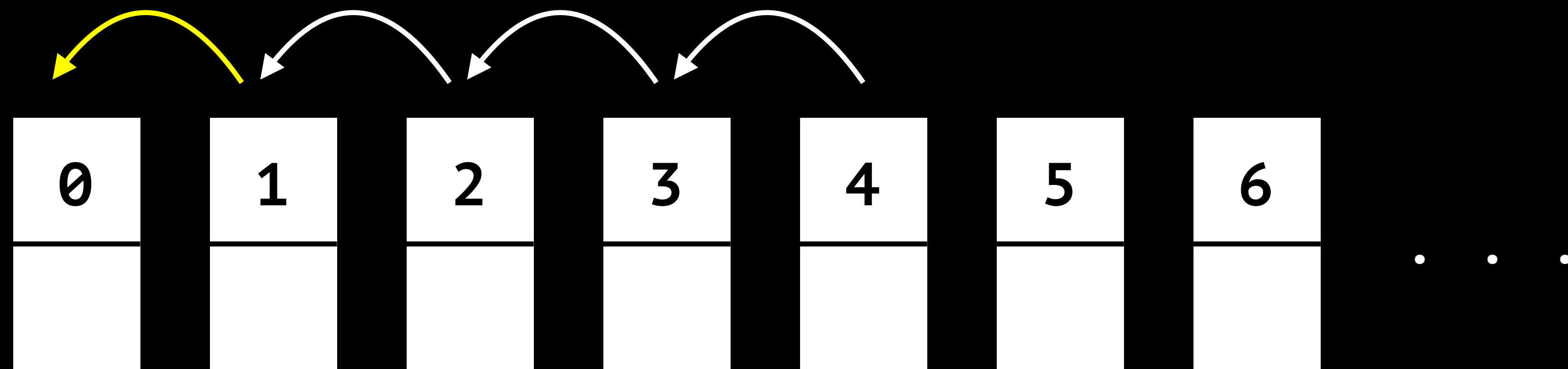

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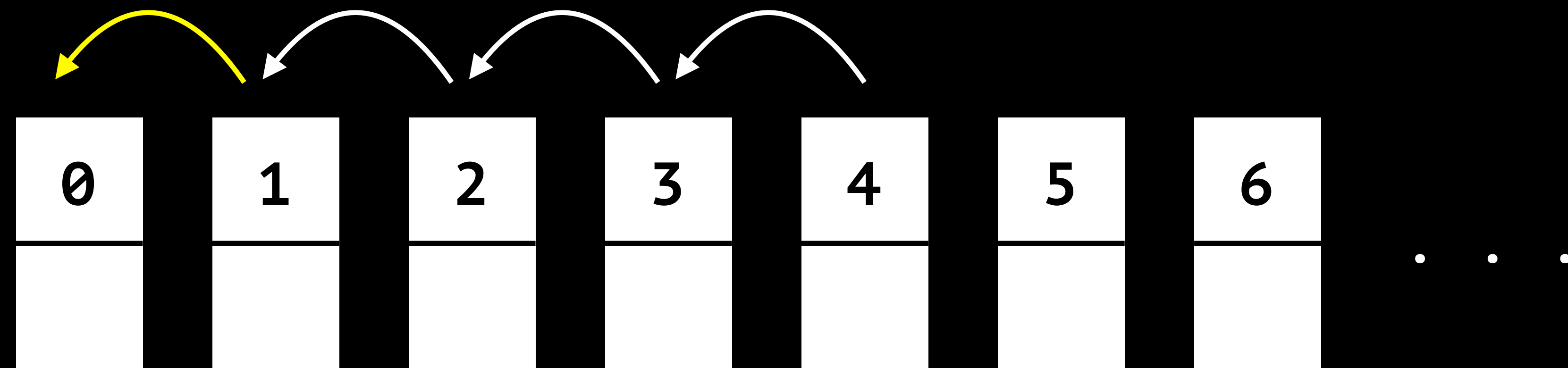

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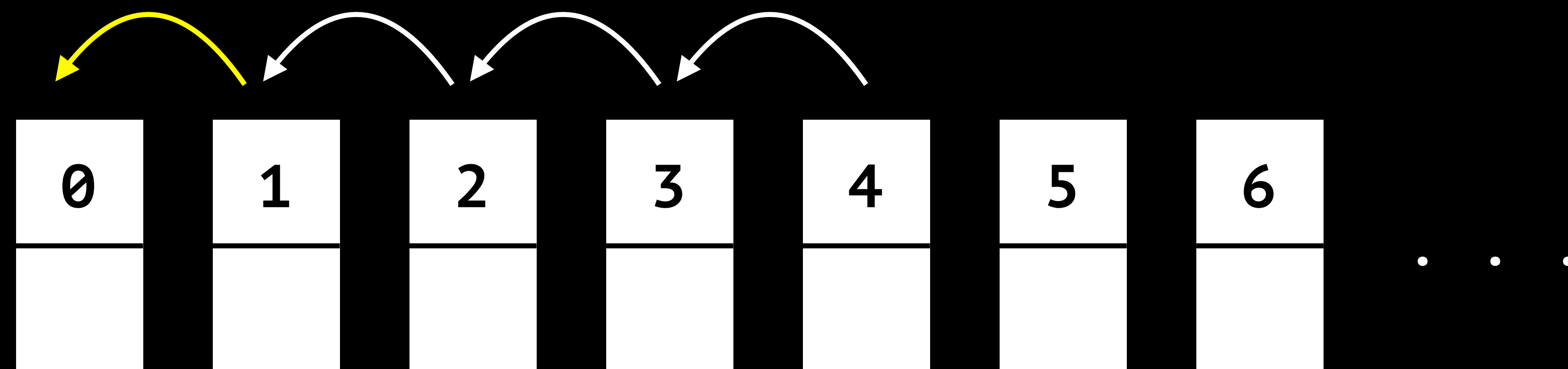

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When you're writing code, assume the recursive call works!

Approaching Recursion

hard functions are hard to write.

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Write triangular.
This computes the nth triangular number, which is 0 + 1 + 2 + … + n

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Recursion, illustrated

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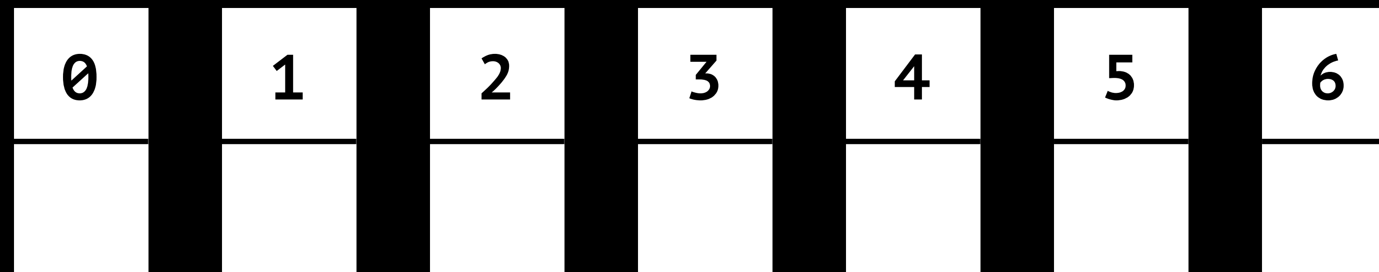
triangular(4) = 4 + triangular(3)

Remember this?



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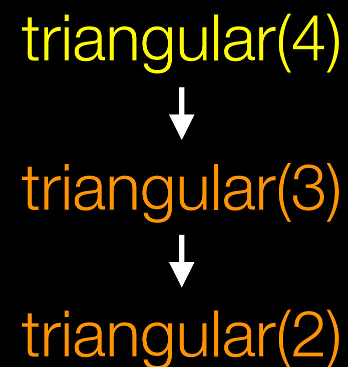

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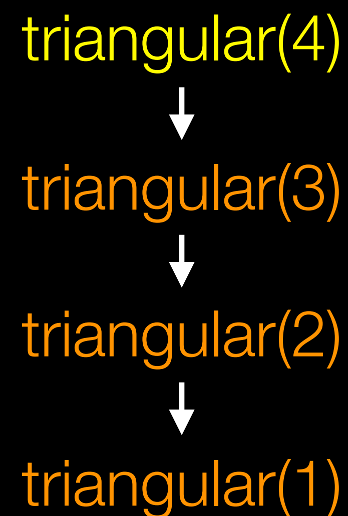

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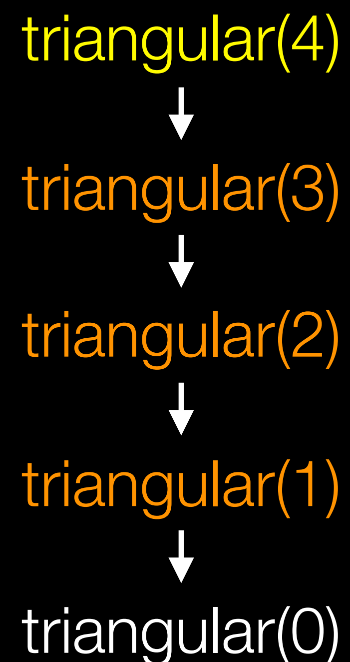

Tree Recursion, illustrated

```
1 def fib(n):
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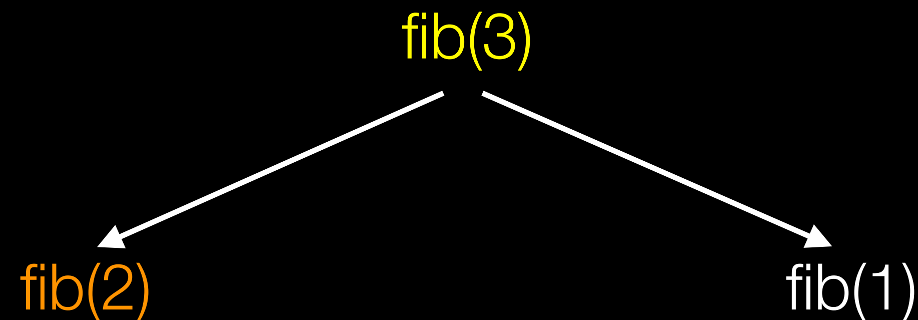

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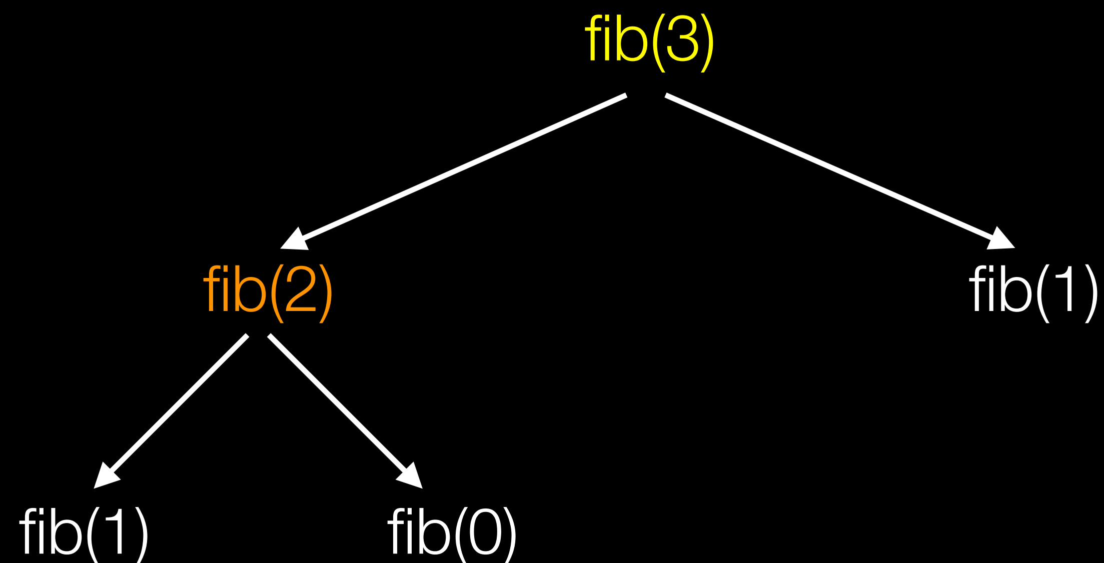

Tree recursion is defined by branching - multiple recursive calls in a frame!

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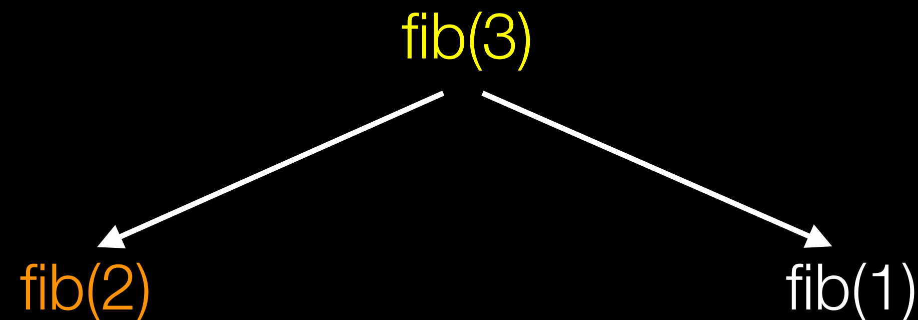

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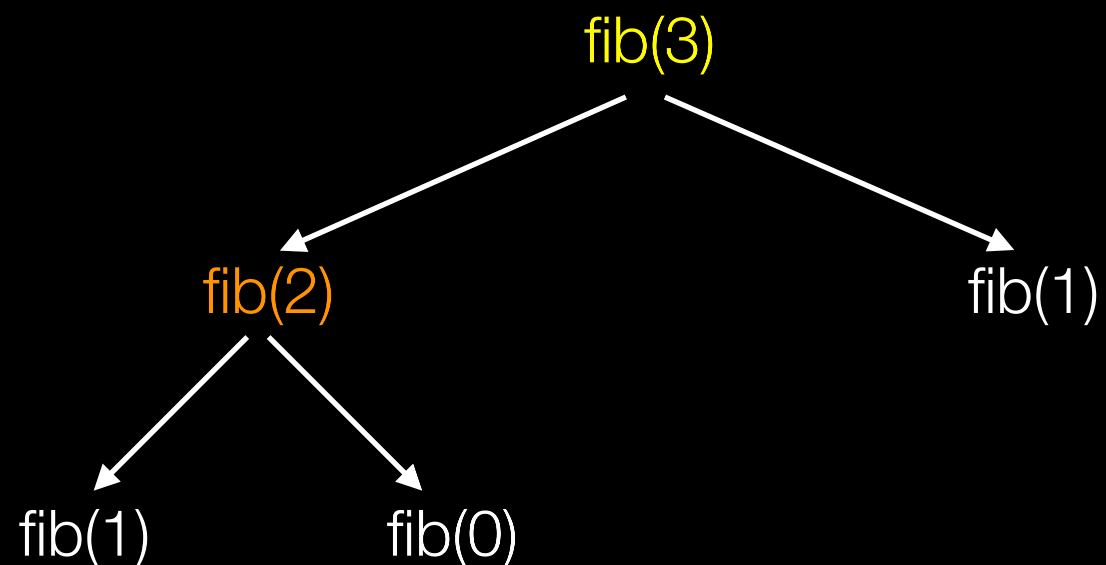

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Tree Recursion

- more recursive calls
- more base cases

But fundamentally this is the same! We're still dealing with dominoes, except sometimes branching.

Thanks for coming.

Have a great rest of your week! :)

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